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Energy Radiation from a Moving Mirror with Finite Mass

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Abstract

In this paper we study energy radiation from a moving mirror in 1+1 dimensional space-time. The mirror is assumed to have finite mass and accordingly to receive back reaction from scalar photon field. The mode expansion of the scalar field becomes different from that without back reaction though the trajectory of the mirror is not changed. Then energy density of the vacuum becomes to have finite value proportional to square of the mass of the mirror. Moreover we compute the energy momentum tensor of the radiation in the case that acceleration of the mirror is small. As a result we show that the mirror creates energy radiation whose quantity does not depend on its mass but on its acceleration even if the acceleration is uniform.

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I. INTRODUCTION

The problem of radiation has entered a new and more interesting phase when it was known that quantum effects on vacuum may extract radiation from the vacuum. Unruh[1] has shown that an accelerated observer observes a heat bath in Minkowski vacuum, which relates to Hawking radiation from a black hole[2] and may give hints on quantum field theory in strong gravitational background. On the other hand, a star collapsing to become a black hole[2] is mimicked by an exponentially accelerated mirror with infinite mass[3]. A geometric boundary made by the mirror gives vacuum a quantum effect same as the collapsing star. Nevertheless these phenomena have right to attract attention even if the relations to black hole is ignored. For example we should consider these effects when studying radiation from electrons in an accelerator[4]. Moreover principle of equivalence should be reinterpreted to contain the quantum effects of radiation.

If we confirm a moving mirror effect experimentally, we will, of course, have to use an object which has finite mass. In this paper we study how a moving mirror with finite mass (say *dynamical* boundary condition for scalar photons) affects on Minkowski vacuum in 1+1 dimensional space-time. We take account of back reaction on the mirror by considering energy and momentum conservation upon reflection of scalar photons[5][6]. Mode functions of the scalar field are accordingly modified into forms which contain integrations by the coordinate and the frequency. In the computation we do not use $(\text{mass})^{-1}$ expansion to see behavior of the out-going wave in ultraviolet region, which is strongly affected by back reaction as easily guessed. As a result energy-momentum tensor of the out-going wave, even the vacuum energy part, becomes finite and energy radiation which is independent of the mass but due to finiteness of the mass appears whenever the mirror is accelerated. It is contrast to the result for the case that a mirror has infinite mass, according to which result radiation does not occur if the acceleration of the mirror is uniform[7].

The remaining sections are organized as follows. In the second section we review the previous result by Fulling and Davies[7] about a moving mirror with infinite mass. In order to clarify relation to the third section, boundary condition is take into account by using Doppler factor. Then we treat a moving mirror with finite mass in the third section. Energy radiation from the mirror is computed for the case that the acceleration of the mirror is small. The last section is assigned to conclusion.

II. A MOVING MIRROR WITH INFINITE MASS

In this section we treat a mirror with infinite mass which travels along a trajectory in two-dimensional Minkowski space-time. The time and space coordinates $\overline{t}$, $\overline{x}$ on the trajectory of the mirror are combined as

$$\overline{x} = z(\overline{t}). \tag{2.1}$$

For simplicity we use for *photon* massless scalar field ϕ which obeys the Klein-Gordon equation

$$\frac{\partial^2 \phi}{\partial u \partial v} = 0, \tag{2.2}$$

where u and v are null coordinates defined as

$$u = t - x, \qquad v = t + x.$$

The scalar field is reflected by the mirror, so that a boundary condition

$$\phi(\bar{t}, \bar{x}) = 0 \tag{2.3}$$

is imposed. Hence it is necessary that the mode expansion is changed from that without boundary condition. First we work in *in mode*, where in-coming wave traveling from the right to the left is expanded with plane waves and out-going wave travelling from the left to the right, on the other hand, may become complicated form. We write the mode function ϕ_{IN} with energy ω as

$$\phi_{IN}(u, v) \sim \exp[-i\omega v] - \exp[-i\omega p(u)], \tag{2.4}$$

where

$$p(u) \equiv 2\bar{t}(u) - u. \tag{2.5}$$

We can similarly obtain the mode function ϕ_{OUT} in *out mode*:

$$\phi_{OUT}(u, v) \sim \exp[-i\omega u] - \exp[-i\omega f(v)], \tag{2.6}$$

where

$$f(v) \equiv 2\bar{t}(v) - v. \tag{2.7}$$

If we regard *light* as classical wave, relation between incident and reflected angular frequencies ω and $\tilde{\omega}_0$ is known as (relativistic) Doppler's formula:

$$\tilde{\omega}_0 = D(V)\omega, \tag{2.8}$$

where velocity of the mirror V and Doppler factor D are

$$V \equiv \frac{dz(t)}{dt}, \tag{2.9}$$

$$D \equiv \frac{\partial \tilde{\omega}_0}{\partial \omega} = \frac{1+V}{1-V}. \tag{2.10}$$

The above two descriptions of the phenomenon that light is reflected by a mirror, of course, relates to each other. Exponents of the mode functions (2.5) and (2.7) are expressed by the Doppler factor D as the following:

$$\begin{aligned} \frac{dp(u)}{du} &= 2\frac{d\bar{t}}{du} - 1 \\ &= \frac{2}{1 - V(u)} - 1 \\ &= D(u), \end{aligned} \tag{2.11a}$$

$$\begin{aligned}\frac{df(v)}{dv} &= 2\frac{d\bar{t}}{dv} - 1 \\ &= \frac{2}{1+V(v)} - 1 \\ &= \frac{1}{D(v)},\end{aligned} \tag{2.11b}$$

Thus

$$\phi_{IN} \sim \exp[-i\omega v] - \exp[-i\omega \int D(u)du], \tag{2.12a}$$

$$\phi_{OUT} \sim \exp[-i\omega u] - \exp[-i\omega \int \frac{dv}{D(v)}]. \tag{2.12b}$$

In this way we obtain the mode functions written by means of the Doppler factor, which functions are useful when back reaction on the mirror is considered.

When the mode function is different from that of plane wave, radiation may occur. We use energy-momentum tensor to investigate the radiation from the moving mirror. The Lagrangian density of the free scalar field in the $t-x$ coordinate system is

$$L = \frac{1}{2}\partial_\mu\phi(t,x)\partial^\mu\phi(t,x). \tag{2.13}$$

Then classical energy-momentum tensor defined as

$$\mathrm{T}^\mu{}_\nu = \frac{\partial L}{\partial(\partial_\mu\phi)}\partial_\nu\phi - \delta^\mu{}_\nu L, \tag{2.14}$$

has components

$$\begin{aligned}\mathrm{T}_{00} &= \mathrm{T}_{11} \\ &= \frac{1}{2}\{(\partial_0\phi)^2 + (\partial_1\phi)^2\}\end{aligned} \tag{2.15a}$$

$$\begin{aligned}\mathrm{T}_{01} &= \mathrm{T}_{10} \\ &= \frac{1}{2}\{\partial_0\phi\partial_1\phi + \partial_1\phi\partial_0\phi\}.\end{aligned} \tag{2.15b}$$

After this we shall regard ϕ as a quantum field operator expanded in *in mode* which is defined as

$$\phi = \int_0^\infty d\omega[a_\omega\phi_\omega + a_\omega^\dagger\phi_\omega^*], \tag{2.16}$$

where a_ω and a_ω^* are annihilation and creation operators, respectively and ϕ_ω is the mode function defined in (2.12a). Since in the above mode expansion the concept of *particle* is clear only for in-coming wave, the *in-vacuum* $\mid 0\rangle$ which is defined as

$$a_\omega \mid 0\rangle = 0 \tag{2.17}$$

should be regarded as the vacuum with no in-coming particle. On the other hand, out-going particles may exist. If one wants to count number of the out-going particles, he or she can do it by, for example, computing Bogoliubov coefficients between in mode and out mode[8]. But relation between the Bogoliubov coefficients and count of particles by a detector is not so clear for some cases[9][10][11]. We will comment on this problem in the last section.

Then the components of the corresponding energy-momentum tensor operator $T_{\mu\nu}(t,x)$, which is obtained by substituting (2.16) into (2.15), are

$$\begin{aligned} T_{00} &= T_{11} \\ &= \tfrac{1}{2}\int_0^\infty d\omega \int_0^\infty d\omega' \; [a_\omega a_{\omega'}\partial_\mu\phi_\omega\partial_\mu\phi_{\omega'} + a^\dagger_\omega a^\dagger_{\omega'}\partial_\mu\phi^*_\omega\partial_\mu\phi^*_{\omega'} \\ &\qquad +2a^\dagger_\omega a_{\omega'}\partial_\mu\phi^*_\omega\partial_\mu\phi_{\omega'} + \delta(\omega-\omega')\partial_\mu\phi^*_\omega\partial_\mu\phi_{\omega'}] \end{aligned} \tag{2.18a}$$

$$\begin{aligned} T_{01} &= T_{10} \\ &= \tfrac{1}{2}\int_0^\infty d\omega \int_0^\infty d\omega' \; [\{a_\omega a_{\omega'}\partial_0\phi_\omega\partial_1\phi_{\omega'} + a^\dagger_\omega a^\dagger_{\omega'}\partial_0\phi^*_\omega\partial_1\phi^*_{\omega'} \\ &\qquad +a^\dagger_\omega a_{\omega'}(\partial_0\phi^*_\omega\partial_1\phi_{\omega'} + \partial_1\phi^*_\omega\partial_0\phi_{\omega'}) \\ &\qquad +\delta(\omega-\omega')\partial_1\phi^*_\omega\partial_0\phi_{\omega'}\} \\ &\qquad +\{\ 0 \ \leftrightarrow \ 1\ \}]. \end{aligned} \tag{2.18b}$$

Expectation value of the energy-momentum tensor in the in-vacuum is defined as

$$\langle T_{\mu\nu}\rangle = T_{\mu\nu} - :T_{\mu\nu}:, \tag{2.19}$$

and thus

$$\langle T_{00}\rangle = \langle T_{11}\rangle = \frac{1}{2}\int_0^\infty d\omega \Big\{(\partial_0\phi_\omega)(\partial_0\phi^*_\omega) + (\partial_1\phi_\omega)(\partial_1\phi^*_\omega)\Big\}, \tag{2.20a}$$

$$\langle T_{01}\rangle = \langle T_{10}\rangle = \frac{1}{2}\int_0^\infty d\omega \Big\{(\partial_0\phi_\omega)(\partial_1\phi^*_\omega) + (\partial_1\phi_\omega)(\partial_0\phi^*_\omega)\Big\}. \tag{2.20b}$$

These expectation values are known to diverge quadratically for the mode function (2.12a). Hence it is necessary to regularize them[8] to obtain physical quantities, which problem is however out of our scope here. We show only the result according to Fulling and Davies[7]:

$$\langle T_{00}(u)\rangle = -\langle T_{01}(u)\rangle = \frac{1}{12\pi}\Big\{D(u)\Big\}^{1/2}\frac{d^2\{D(u)\}^{-1/2}}{du^2}, \tag{2.21}$$

where a constant quadratically divergent term has been discarded. The renormalized expectation value (2.21) describes radiation from the mirror with no in-coming scalar particles and it vanishes when acceleration of the mirror is uniform.

III. BACK REACTION

If one regards a moving mirror simply as a model of a collapsing star, the mirror should have infinite mass to fix its trajectory; in other words the boundary condition for the scalar photon field should be geometric. Nevertheless, if one regards a moving mirror as a real

object and wants to confirm its effect experimentally, he or she should take account of back reaction on the mirror. In this section we study how the back reaction on the mirror with finite mass m affects energy radiation. We let the mirror particle and a massless scalar field obey energy-momentum conservation law at elastic scattering between them. It is not expected however that the scalar particle really collides with the mirror since we will compute physical quantities in *in vacuum*, which contains no in-coming particles. Hence, the trajectory of the mirror is left unchanged though the mode function of the out-going wave is affected by the dynamical boundary condition.

The energy and momentum conservation at the collision lead the following equations:

$$m\gamma + \omega = m\tilde{\gamma} + \tilde{\omega}, \tag{3.1a}$$

$$-m\gamma V + \omega = -m\tilde{\gamma}\tilde{V} - \tilde{\omega}, \tag{3.1b}$$

where $\tilde{V}$ and $\tilde{\omega}$ are velocity of the mirror and energy of the scalar field after the collision, respectively, and

$$\gamma \equiv \frac{1}{\sqrt{1-V^2}}\ , \qquad \tilde{\gamma} \equiv \frac{1}{\sqrt{1-\tilde{V}^2}}.$$

Regarding $\tilde{\omega}$ and $\tilde{V}$ as functions of ω and V, we obtain Doppler factor $\tilde{D}$ with back reaction

$$\begin{aligned}\tilde{D} &\equiv \frac{\partial\tilde{\omega}}{\partial\omega} \\ &= \frac{1+\tilde{V}}{1-\tilde{V}} \\ &= \left\{D^{-1/2} + \frac{2\omega}{m}\right\}^{-2}.\end{aligned} \tag{3.2}$$

If we take $m \to \infty$, $\tilde{D}$ becomes equal to D. For finite mass and large ω, however, $\tilde{D}$ is small and energy of the reflected particle is reduced severely. It induces convergence of energy-momentum tensor of the out-going wave, as we shall see later.

Then the mode functions (2.12) will be modified into

$$\phi_{IN} = A\exp[-i\omega v] - B\exp[-i\int\int \tilde{D}(u)d\omega du], \tag{3.3a}$$

$$\phi_{OUT} = A'\exp[-i\tilde{\omega}u] - B'\exp[-i\int\int \frac{d\tilde{\omega}dv}{\tilde{D}(v)}], \tag{3.3b}$$

where A, A', B and B' are normalization factors.

Here for convenience we introduce new notations $\tilde{\omega}(\omega)$ and $\omega(\tilde{\omega})$, which are solutions of equations (3.1), defined as

$$\begin{aligned}\tilde{\omega}(\omega) &\equiv \int \tilde{D}(u)d\omega \\ &= -\frac{m}{2}(D^{-1/2} + \frac{2\omega}{m})^{-1} + \frac{mD^{1/2}}{2},\end{aligned} \tag{3.4a}$$

$$\omega(\tilde{\omega}) \equiv \int \frac{d\tilde{\omega}}{\tilde{D}(v)}$$
$$= +\frac{m}{2}(D^{1/2} - \frac{2\tilde{\omega}}{m})^{-1} - \frac{mD^{-1/2}}{2}. \tag{3.4b}$$

Thus

$$\phi_{IN} = A\exp[-i\omega v] - B\exp[-i\int \tilde{\omega}(\omega)du], \tag{3.5a}$$

$$\phi_{OUT} = A'\exp[-i\tilde{\omega}u] - B'\exp[-i\int \omega(\tilde{\omega})dv]. \tag{3.5b}$$

These modified forms of the mode functions are natural extension of (2.12) because the out-going (in-coming) wave of in (out) mode, as expected, becomes plane wave with the appropriate energy and momentum if velocity of the mirror is constant.

To discuss orthonormality condition of the functions (3.3a), let us compute the Klein-Gordon inner product of the out-going wave without the normalization factor B.

$$\begin{aligned}
&(\exp[i\int \tilde{\omega}(\omega_1)du], \exp[-i\int \tilde{\omega}(\omega_2)du]) \\
&= i\int dx\Big\{\exp[i\int \tilde{\omega}(\omega_1)du]\overrightarrow{\partial_t}\exp[-i\int \tilde{\omega}(\omega_2)du] - \exp[i\int \tilde{\omega}(\omega_1)du]\overleftarrow{\partial_t}\exp[-i\int \tilde{\omega}(\omega_2)du]\Big\} \\
&= \int dx(\tilde{\omega}(\omega_1) + \tilde{\omega}(\omega_2))\exp[i\int(\tilde{\omega}(\omega_1) - \tilde{\omega}(\omega_2))]du.
\end{aligned} \tag{3.6}$$

If we recall the definition of $\tilde{\omega}(\omega)$, we obtain two convenient equations:

$$\tilde{\omega}(\omega_1) - \tilde{\omega}(\omega_2) = (\omega_1 - \omega_2)\tilde{D}(\omega_1, \omega_2),$$

$$\tilde{\omega}(\omega_1) + \tilde{\omega}(\omega_2) = \tilde{D}(\omega_1, \omega_2)\Big\{(\omega_1 + \omega_2) + \frac{4D^{1/2}\omega_1\omega_2}{m}\Big\},$$

where

$$\tilde{D}(\omega_1, \omega_2) \equiv \Big(D^{-1/2} + \frac{2\omega_1}{m}\Big)^{-1}\Big(D^{-1/2} + \frac{2\omega_2}{m}\Big)^{-1}. \tag{3.7}$$

These equations enable us to proceed with the computation as

$$\begin{aligned}
(3.6) &= \int dx\tilde{D}(\omega_1, \omega_2)\Big\{(\omega_1 + \omega_2) + \frac{4D^{1/2}\omega_1\omega_2}{m}\Big\}\exp[i(\omega_1 - \omega_2)\int \tilde{D}(\omega_1, \omega_2)]du \\
&= (\omega_1 + \omega_2)\int dX\exp[i(\omega_1 - \omega_2)X] + \frac{4\omega_1\omega_2}{m}\int dX D^{1/2}\exp[i(\omega_1 - \omega_2)X] \\
&= 4\pi\omega_1\delta(\omega_1 - \omega_2) + \frac{4\omega_1\omega_2}{m}\int dX D^{1/2}\exp[i(\omega_1 - \omega_2)X],
\end{aligned} \tag{3.8}$$

where

$$X \equiv \int \tilde{D}(\omega_1, \omega_2)du.$$

The last term in the last line of (3.8), which is due to the finiteness of the mass of the mirror, does not produce Dirac's delta function unless D is constant (no acceleration). If D is constant, the mode expansion is exactly determined. In this case the out-going wave becomes plane wave whose frequency has upper bound $mD^{1/2}/2$ and the energy-momentum tensor of the out-going wave has finite value $Dm^2/32\pi$. We think it appropriate to regard divergence which appears when the mass goes to infinity as discarded constant divergence mentioned in the previous section. Hence the last term in the line of (3.8) gives energy of the vacuum and should be discarded when one computes physical quantities.

In general it is necessary to carry out the integration and let the function satisfy orthonormality condition in order to determine the mode function for each trajectory of a mirror. Here we study the case that acceleration of the mirror is small, so that the mode function in in mode is approximately given by (3.3a) with *function B*:

$$B^{-2}(u) = 2\pi\Big\{2\omega + \frac{4\omega^2 D^{1/2}(u)}{m}\Big\}. \tag{3.9}$$

We may expect that the modification on the mode expansions due to the back reaction will cause radiation of the scalar particles. Computing energy-momentum tensor of the out-going wave, we show what effects the back reaction give to the radiation. The mode function of the out-going wave in in mode is written as

$$\begin{aligned}\phi_{IN}^{out-going} &\equiv \varphi \\ &= \frac{-1}{\sqrt{4\pi\omega}}\left(1+\frac{2\omega D^{1/2}(u)}{m}\right)^{-1/2} \exp[-i\int\tilde{\omega}(\omega)du].\end{aligned} \tag{3.10}$$

Then

$$\left.\begin{matrix}\partial_0\varphi \\ \partial_1\varphi\end{matrix}\right\} = \frac{\pm 1}{\sqrt{4\pi\omega}}\left(1+\frac{2\omega D^{1/2}}{m}\right)^{-3/2}\left\{\frac{\omega D^{-1/2}\dot{D}}{2m} + i\Big(1+\frac{2\omega D^{1/2}}{m}\Big)\tilde{\omega}\right\}\exp[-i\int\tilde{\omega}du], \tag{3.11a}$$

$$\left.\begin{matrix}\partial_0\varphi^* \\ \partial_1\varphi^*\end{matrix}\right\} = \frac{\pm 1}{\sqrt{4\pi\omega}}\left(1+\frac{2\omega D^{1/2}}{m}\right)^{-3/2}\left\{\frac{\omega D^{-1/2}\dot{D}}{2m} - i\Big(1+\frac{2\omega D^{1/2}}{m}\Big)\tilde{\omega}\right\}\exp[+i\int\tilde{\omega}du], \tag{3.11b}$$

where the dot means differentiation by u. Substituting them into (2.20), we obtain the energy-momentum tensor of the out-going wave:

$$\begin{aligned}\langle T_{00}\rangle = \langle T_{11}\rangle &= -\langle T_{01}\rangle = -\langle T_{10}\rangle \\ &= \int_0^\infty d\omega\frac{1}{4\pi\omega}\left(1+\frac{2\omega D^{1/2}}{m}\right)^{-3}\left(\frac{\omega^2\dot{D}^2}{4m^2D}+\omega^2D^2\right) \\ &= \frac{1}{16\pi}\left(\frac{Dm^2}{2}+\frac{\dot{D}^2}{8D^2}\right) \\ &= \frac{Dm^2}{32\pi}+\frac{\gamma^2\alpha^2}{32\pi},\end{aligned} \tag{3.12}$$

where

$$\alpha \equiv \gamma \dot{V}.$$

In this calculation no regularization is needed since the high frequency part is strongly suppressed by recoil of the mirror. The first term in the last line of (3.12) is, as previously mentioned, energy-momentum tensor for out-going plane wave. The second term does not appear if we put the mass of the mirror infinite from the beginning of the computation, though it is independent of the mass. We regard this term as energy radiation from the mirror when no in-coming photon exist.

IV. CONCLUSION

As well known, boundaries may extract radiation from vacuum. In this paper we have treated the boundary made of a particle with finite mass (dynamical boundary), which particle obeys energy and momentum conservation law when it collides with a virtual scalar photon. Mode expansion of the photon is accordingly changed to have upper bound of its energy. Moreover the dynamical boundary causes energy radiation independent of the mass and proportional to square of acceleration of the mirror. Such effects are not perturbative compensation for geometrical boundary condition but novel aspects of the dynamical one. Thus in some cases it may be appropriate and essential to impose dynamical boundary condition instead of geometric one.

When a detector is introduced to count number of particles, we should similarly remember that the detector has finite mass. For example a rotating detector in Minkowski vacuum may response though the rotating vacuum is the same. This is not a quantum effect but a rather trivial one: Radiation are caused using energy which the detector has lost with recoil. It does not vanish even if mass of the detector is thought to be infinity at the end of the computation[12].

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